**Differentiation And Integration**

In the last lesson you learned how we can go from position to velocity to acceleration by "differentiating" or "taking the derivative".

In this lesson, you'll learn how to go from acceleration to velocity to position using something called an "integral".

# Differentiation Recap

In the last lesson, you learned about the **derivative**. This section is just here to remind you of what you learned.

### Understanding the Derivative

You saw a few ways to understand the derivative:

##### 1. The "Rate of Change" Interpretation

If f(t)*f*(*t*) gives the value of a function at **any** t*t*, then \dot{f}(t\_0)*f*˙​(*t*0​) gives the instantaneous rate of change of f(t)*f*(*t*) at the value t=t\_0*t*=*t*0​.

##### 2. The Graphical Interpretation

The slope of the line tangent to f(t)*f*(*t*) at t=t\_0*t*=*t*0​ is \dot{f}(t\_0)*f*˙​(*t*0​)

A close up of a map

Description automatically generated

The slope of the orange line is equal to the **derivative** of f at t=t\_0

##### 3. The Formal Definition

The formal mathematical definition is the following:

The **derivative** of a function f(t)*f*(*t*) is the function \dot{f}(t)*f*˙​(*t*) (or \frac{df}{dt}*dtdf*​), and is defined as:

\dot{f}(t) = \lim\_{\Delta t \to 0}\frac{ f(t + \Delta t) - f(t)}{\Delta t}*f*˙​(*t*)=Δ*t*→0lim​Δ*tf*(*t*+Δ*t*)−*f*(*t*)​

### Derivatives and Motion

**Position**, **velocity**, and **acceleration** are all useful quantities when describing a vehicle's motion and these quantities are related through the derivative.

* velocity is the derivative of position
  + v(t)=\dot{x}(t)*v*(*t*)=*x*˙(*t*)
* acceleration is the derivative of velocity and the second derivative of position.
  + a(t) = \dot{v}(t) = \ddot{x}(t)*a*(*t*)=*v*˙(*t*)=*x*¨(*t*)

### Coding the Derivative

**def** **get\_derivative\_from\_data**(position\_data, time\_data):

"""

Calculates a list of speeds from position\_data and

time\_data.

Arguments:

position\_data - a list of values corresponding to

vehicle position

time\_data - a list of values (equal in length to

position\_data) which give timestamps for each

position measurement

Returns:

speeds - a list of values (which is shorter

by ONE than the input lists) of speeds.

"""

*# 1. Check to make sure the input lists have same length*

**if** len(position\_data) != len(time\_data):

**raise**(ValueError, "Data sets must have same length")

*# 2. Prepare empty list of speeds*

speeds = []

*# 3. Get first values for position and time*

previous\_position = position\_data[0]

previous\_time = time\_data[0]

*# 4. Begin loop through all data EXCEPT first entry*

**for** i **in** range(1, len(position\_data)):

*# 5. get position and time data for this timestamp*

position = position\_data[i]

time = time\_data[i]

*# 6. Calculate delta\_x and delta\_t*

delta\_x = position - previous\_position

delta\_t = time - previous\_time

*# 7. Speed is slope. Calculate it and append to list*

speed = delta\_x / delta\_t

speeds.append(speed)

*# 8. Update values for next iteration of the loop.*

previous\_position = position

previous\_time = time

**return** speeds

I exaggerated in the video when I said the peaks look like mirror images. In fact the first peak is wider but not as tall (in absolute value) while the second peak is "peakier".

But that just makes it even more amazing that the area under each of these curves still winds up being equal!

A close up of a map

Description automatically generated

### Finding Area Practice

The following four questions are based on graphs of vehicle velocity vs time. For these graphs the horizontal axis is in units of **seconds** and the vertical axis is in units of **meters / second**.

A picture containing cow, sitting, looking, group

Description automatically generated

A picture containing looking, colored, sitting, light

Description automatically generated

A picture containing crossword, cow, group, couple

Description automatically generated

A picture containing looking, group, sitting, couple

Description automatically generated

A car's heading is usually represented by the Greek letter theta: \theta*θ*

The *angular velocity* is the rate of change (derivative) of the heading, so it's represented by "theta dot": \dot{\theta}*θ*˙

Just so you know: not *all* of the data I've shown you has been fake. The elevator data at the beginning of the lesson was 100% real!

As you just saw, it can be dangerous to rely on accelerometer data for localization since errors have a tendency to accumulate. This is a weakness of accelerometers.

Fortunately, it takes some time for these errors to accumulate. So when they're used over short time intervals accelerometers can be really helpful.

Take a look at one of Uber's early prototypes of a self driving car:

A car driving on a city street

Description automatically generated

Look at all those sensors perched on the hood! And you can't even see the IMUs and odometers and GPS sensors inside the vehicle!

Each of these sensors has strengths and weaknesses and each contributes to an improved understanding of the vehicle and it's surroundings.

If you choose to take the advanced Self Driving Car Engineer Nanodegree, you'll learn about **sensor fusion**. Sensor fusion is how you use software to stitch together all these disparate data sources into one coherent picture about the vehicle, its motion, and the world around it.

Congratulations! Next up: trigonometry!